A Failure of the No-Arbitrage Principle*

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Abstract

Underlying the principle of no arbitrage is the assumption that markets eliminate any opportunity for risk-free profits. In contrast, we document a pricing mistake by a $200 million company that allowed investors a guaranteed return of 25.6% in a few days, and that resulted in less than $60,000 being invested into exploiting the opportunity.

1 Introduction

The principle of no arbitrage—that prices cannot allow for risk-free net profits—plays a key role in the understanding of prices and price movements in financial markets, being a central component of such fundamental concepts and theories as the efficient-markets hypothesis, arbitrage pricing theory, and option pricing. The principle does not require that every individual in the market be fully rational and prefer more money to less, but it does require that sufficiently many motivated decisionmakers with access to sufficient resources notice any possible mispricing, put a lot of pressure on it, and quickly eliminate it. Questioning part of this argument, recent research on the "limits of arbitrage" has emphasized that even if investors notice a mispricing, they may not be able to or want to fully correct it.[1] But very little research has addressed the other necessary ingredient

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[1] For a summary of this research and a detailed discussion of arbitrage, see for instance Shleifer (2000). For examples of theories of limited arbitrage, see De Long, Shleifer, Summers, and Waldmann (1990) and Shleifer and

of the no-arbitrage principle, that sufficiently many people will notice and exploit mispricing when doing so allows for large riskless profits independently of one's subjective beliefs.

In this note we document an instance in which one firm, gambling monopoly Szerencsejáték Rt. (SzjRt) of Hungary, created a striking arbitrage opportunity against itself, and survived it with flying colors. In one week of peak demand, SzjRt's pricing structure—which was widely read public information—allowed an arbitrage opportunity very similar to those underlying derivative pricing: bettors could construct a portfolio of correlated assets leaving them perfectly hedged, and make a return of over 25% in a couple of days.[2] Figure 1 shows SzjRt's weekly profits in 1998 from the game in which arbitrage was possible. The arbitrage opportunity occurred in Week 26, in which SzjRt posted the year's *highest* profits from the game! Clearly, even in this ideal setting for arbitrage, the hypothesis that sufficiently many individuals notice and exploit mispricing fails. Indeed, we calculate that the (very conservative) upper bound on the amount of money that could have been placed as part of an arbitrage strategy is 12,511,000 HUF (about $59,500 at the time), a mere 5.46% of revenue and 11% of payouts from that single game that week. This is so little money that it must have been a minor headache for SzjRt that week. In fact, if it had any other reason for the pricing structure in place—for instance, to make the game more exciting in a week of peak demand—the company may, instead of going bankrupt as the principle of no arbitrage predicts, have made money on this "mis"pricing.

2 Some Background

Szerencsejáték Rt. may be an unusual target for arbitrage, but if anything it is a safer bet for investors than most targets. The company is Hungary's state-owned monopoly provider of number-draw games and sports bets. With revenue of just under 44 billion HUF (about $210 million) in 1998, it is one of the largest companies in the country. Its profits are absorbed into, and its

Vishny (1997). Lamont and Thaler (2003) give examples of very large deviations from the law of one price that were apparently widely known; they argue that the limits of arbitrage were so great that even large mispricing could not be corrected.

[2] Interested readers can—instead of reading our explanation below—try to find the arbitrage opportunity themselves by opening and looking at (our translation of) what gamblers saw. We conjecture that even with the substantial benefits of a very high IQ and of the knowledge that there *is* an opportunity, readers will not find it immediately apparent. In fact, one author, although he was a regular player of the game at the time, failed to spot the opportunity.

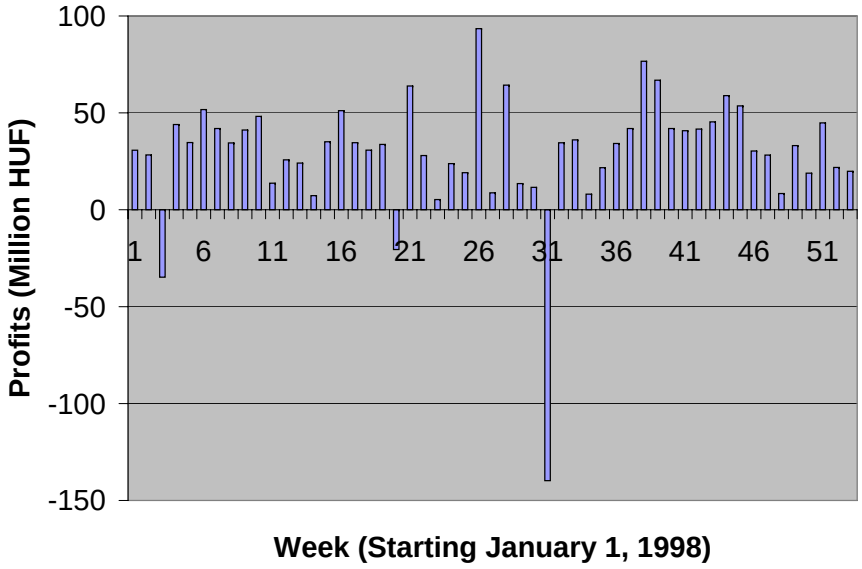

Figure 1: Szerencsejáték Rt's Weekly Profits From the Tippmix Game in 1998. (Note: The $-HUF exchange rate at the time was 210.)

potential losses are backed by, the state budget. Because SzjRt therefore has no option to go bankrupt, its promises to pay are more credible than those of most private companies, regarding which the question of arbitrage usually arises. In addition, while SzjRt reserves the right to cancel sports bets, the 1991 Act on the Organization of Gambling Operations allows it do so only when the actual sports event is canceled or there is a case of documentable corruption. Because the arbitrage opportunity is based on matches in soccer's biggest international event, the World Cup, there is essentially no chance that any of these would happen.

The relevant details of the game in question, Tippmix, were public information in any sense of the word commonly understood in economics and finance. The betting opportunities were published in SzjRt's sports-gambling newsletter *Sportfogdás* (circulation: 100,000), the sports daily *Nemzeti Sport* (which is read by most sports fans), on Teletext, at several hundreds of SzjRt outlets, and at almost all post offices in Hungary. Even considering how many people actually *read* the information—a measure that seems stricter than that typically applied for the publicness of information—details of Tippmix were widespread. Conservative estimates of SzjRt indicate that an

#	game	former	tie	latter
8.	Chicago-Dallas	1.50	3.00	4.00
9.	Ivanchuk-Kramnik	3.20	1.50	3.65

Table 1: Two Bettable Partitions from Week 27, 1998

average of at least 10,000 people played each week. Since revenue in our week of interest was way above the year's average (229,032,600 HUF versus 111,128,321 HUF), it is likely that the actual number of players was higher still. Furthermore, while Tippmix was targeted at a general audience, unpublished market studies of SzjRt suggest that it drew much interest from the wealthier, college-educated part of the population.

3 The Opportunity

The arbitrage opportunity we analyze arose in the Tippmix game offered by SzjRt. Tippmix is a relatively complicated sports gambling game, as it gives individuals substantial flexibility to make "combined bets." For example, in Week 27 in 1998 gamblers could combine bets on the outcomes of the Ivanchuk-Kramnik chess match in Germany and the Chicago-Dallas soccer match in the United States. Table 1 shows part of the two lines corresponding to these bets in the list of 114 possible bets that week. Each outcome of each match has an "odds" specifying the amount of money one can win when betting on that outcome alone. For example, the odds for Dallas winning the Chicago-Dallas game was 4, meaning that if an individual placed a singleton bet of 1HUF on this outcome and she got it right, she would have received 4HUF. If a person makes a combined bet on multiple matches, she wins money only if all her guesses are right; but in that case, the relevant multiplier is the product of the odds of the individual outcomes. Hence, for example, if someone put a "double bet" on Dallas and Ivanchuk winning and was right, she would have received $4 \times 3.2 = 12.8$ times her wager.

The fact that bettors could place combined bets does not in itself mean that arbitrage opportunities easily arise. The reason this was possible is that in Week 26 in 1998, Szerencsejáték

#	game	former	tie	latter
8.	Ivanchuk1-Kramnik1	3.20	1.50	3.65
9.	Ivanchuk2-Kramnik2	3.20	1.50	3.65

Table 2: Hypothetical Example with Logically Connected Bets

Rt. allowed gamblers to combine multiple bets on the *same* sports event. To illustrate the implications in an extreme example, suppose that bettors could combine two logically identical bets on the outcome of the actual Ivanchuk-Kramnik match, as illustrated in the hypothetical Table 2. Then, to win 1HUF if Ivanchuk wins, it suffices to place a double wager of $1/(3.2 \times 3.2)$ on both Ivanchuk1 and Ivanchuk2 winning (recall that the odds from different bets are multiplied). Extending this logic, to win 1HUF for any outcome of the match, it suffices to spend $1/(3.2 \times 3.2) + 1/(1.5 \times 1.5) + 1/(3.65 \times 3.65) \approx 0.62$HUF. Hence, while making a singleton bet cannot generate riskless profit, this double bet yields a whopping riskless return of 61%! Why does this work? Intuitively, if a combined bet is drawn from different matches, to make sure one wins one must bet on all combinations of outcomes. But if bets are logically related because they derive from the same match, it may take much fewer wagers to cover all possible outcomes of the match.

The actual arbitrage opportunity was of course more complicated because the possible related bets were not identical, and because SzjRt had restrictions in place on the minimum number of bets that had to be involved in a combined bet. The most profitable balanced arbitrage strategy—where the amount of winnings is riskless—is based on the Argentina-Croatia game in Week 26.[3] Gamblers could bet on whether the game would be a win, tie, or loss for Argentina, as well as on what the score would be (0-0, 1-0, 0-1, etc., or "other"). Any exact score fully determines whether the game is a win, tie, or loss, so the two bets are very closely linked logically. To construct an arbitrage, we combine each score with the appropriate summary outcome, and the "other" score with all three possibilities, covering all possible outcomes of the match. A difficulty is that for these highly arbitrageable bets gamblers had to place at least five-fold combined bets. But it just

[3] There were also two less profitable balanced arbitrage strategies and of course countless non-balanced arbitrage strategies.

so happens that SzjRt allowed another pair of related bets: whether Yugoslavia would win, tie, or lose its match with the United States, and whether it would win by at least 2 goals, 1 goal, or not win. As our fifth (or possibly sixth) bet, we use the number of goals scored by either Suker (a Croat) or Batistuta (an Argentine) or both, since if either Croatia or Argentina is scoreless, that automatically implies their player does not score. The details of this arbitrage strategy are in the appendix, where we show that it produced a risk-free return of 25.6% in just a few days.[4]

4 Evidence on Responses

Even based on aggregate evidence, it is immediately apparent that exploitation of the arbitrage opportunity could not have been widespread. As shown in Figure 1, SzjRt realized a record profit from Tippmix during the week in question. While this record profit was partly due to record demand for Tippmix, it was also due to a 41% profit rate that was well above the year's average of 26%. In contrast, if everyone had followed the most profitable balanced arbitrage strategy, then SzjRt would have realized a loss of 25%.

Beyond these aggregate measures, we can provide a rough upper bound on the amount of money that could have been invested into arbitrage strategies. For each possible bet, we have obtained from SzjRt the total amount of money that was spent in wagers involving that bet (including singleton bets and combined bets of all sizes). Hence, we exploit that certain bets had to be involved in any arbitrage strategy. As we show in the appendix, any arbitrage strategy must involve bets that are logically connected. In Week 26 there were exactly three matches on which multiple bets were allowed: the Yugoslavia-US, Argentina-Croatia, and Germany-Iran matches of the World Cup. Based on simple but tedious calculations, we show in the appendix that any arbitrage had to involve the Argentina-Croatia match and at least one of the other two matches.

Based on this very rough consideration, we get a very low upper bound for the amount of arbitrage money in Tippmix. Consumers spent 5,235,000 HUF ($25,000) betting on whether Ger-

[4] Implementing an arbitrage strategy involved modest but non-trivial transaction costs because it required filling out many tickets. For example, implementing the most profitable balanced arbitrage above would have required filling out 116 tickets, which we estimate would have taken one to two hours. This work was, however, largely a fixed cost: once the tickets were filled out, a bettor could increase the stakes of the bet in principle without limit.

many wins by 2 goals, 1 goal or does not win against Iran, and 7,276,000 HUF ($34,500) betting on whether Yugoslavia wins by 2 goals, 1 goal or does not win against the US. This means that no more than 12,511,000 HUF ($59,500), a mere 5.46% of the wagers in Tippmix, was spent on arbitrage strategies. Furthermore, our data suggests that much or most of even this money may not have been coming from arbitrageurs. In Week 26 the amount placed on "Belgium wins its soccer match by 2 goals/1 goal/does not win" was 8,902,000 HUF ($42,500)—higher than the amount placed on either arbitrage bet of the same form. Similarly, in nearby weeks where there was no arbitrage opportunity, the amount of money placed on bets of the form "Team X wins its soccer match by 2 goals/1 goal/does not win" varied between 3% and 10% of revenue. A combined 5.46% on two such bet is on the low side of this range of non-arbitrage demand.

5 Conclusion

Our note is only the case study of a single instance where market participants failed to exploit a very profitable arbitrage opportunity. In itself, it answers neither why most individuals failed to spot the opportunity, nor how common unexploited arbitrage opportunities are. To gain traction on these questions, a systematic analysis of a large number of arbitrage opportunities would be useful.

References

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990): "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98(4), 703–738.

Lamont, O., and R. H. Thaler (2003): "Can the Stock Market Add and Subtract? Mispricing in Tech Stock Carve-Outs," *Journal of Political Economy*, 111(2), 227–268.

Shleifer, A. (2000): *Clarendon Lectures: Inefficient Markets*. Oxford University Press.

Shleifer, A., and R. W. Vishny (1997): "The Limits of Arbitrage," *Journal of Finance*, 52(1), 35–55.