Appendices 2 and 3 to

A Theory of Reciprocity

by

Armin Falk and Urs Fischbacher

7 Appendix 2: Extensions

7.1 Extension to Games With More Than 2 Players

The idea behind the generalization to games with more than 2 players consists of considering the reciprocity relation of player i independently towards each of the other players $j \neq i$.

Let $s_i \in S_i$ be player i's behavior strategy and let $s_i^{(j)} \in S_j$ be player i's first order belief about player j's strategy. Further let $s_i^{(jk)} \in S_k$ be player i's (second order) belief about what he thinks is player j's belief about k's strategy. In the 2 player case, we get $s_i' = s_i^{(j)}$ and $s_i'' = s_i^{(ji)}$. Furthermore, we use the notation $(-ij)$ to express the set of players other than players i and j. So, $s_i^{(j(-ij))}$ is player i's belief about what j believes what the other players will do. In analogy to the definitions (2) to (6) we define in a node n - for a given set of first and second order beliefs - the following expressions:

$$\Delta_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) := \pi_i(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) - \pi_j(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \tag{11}$$

(In the two player case $\Delta_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) = \Delta_j(n, s_i'', s_i')$ holds.)

$$\Pi_i^j(n, s_i^{(ji)}, s_i^{(j(-ij))}) := \left\{ \left(\pi_i(s_i^{(ji)}|n, s_j^p, s_i^{(j(-ij))}|n), \pi_j(s_i^{(ji)}|n, s_j^p, s_i^{(j(-ij))}|n)\right) \mid s_j^p \in S_j^p \right\} \tag{12}$$

$$\begin{aligned} &\vartheta_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) := \\ \max &\Big\{ \Omega(\tilde{\pi}_i, \tilde{\pi}_j, \pi_i(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}), \pi_j(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))})) \mid \\ &(\tilde{\pi}_i, \tilde{\pi}_j) \in \Pi_i^j(n, s_i^{(ji)}, s_i^{(j(-ij))}) \Big\} \end{aligned} \tag{13}$$

$$\varphi_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) = \vartheta_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \Delta_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \tag{14}$$

$$\sigma_{i\to j}(n, f, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) := \pi_j(\nu(n,f), s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) - \pi_j(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \tag{15}$$

(In the two player case $\sigma_{i\to j}(n, f, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) = \sigma_i(n, f, s_i'', s_i')$ holds.)

$$\begin{aligned} &U_i(f, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) = \\ = \pi_i(f) + \rho_i \textstyle\sum_{j\neq i} \sum_{n\in N_i}^{n\to e} &\varphi_{j\to i}(n, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \sigma_{i\to j}(n, f, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) \end{aligned} \tag{16}$$

A reciprocity equilibrium is again a set of strategies and first and second order beliefs such that the strategies maximize the utility in Equation (**??**) and such that strategies and beliefs are consistent.

7.2 Extension to Games with Almost Perfect Information

Our model can easily be extended to games with almost perfect information which is a class of games that includes games with perfect information as well as simultaneous move games. Games with almost perfect information are multi-stage games with perfect monitoring. This means that players move simultaneously and after each move, all players' actions are revealed and the next move starts. A game with almost perfect information can be visualized with a game tree in which nodes are histories. In each history the players choose simultaneously and the links to the next history are all possible action combinations in this history (cardinal product of the action spaces in the history). A game with perfect information is a special case of a game with almost perfect information in which in every history at most one player has more than one action to choose from. A simultaneous move game is a special case of a game with almost perfect information in which there is only one history in which players have an action to choose from.

Our model can easily be generalized to this class of games. The expected payoff conditional on a node trivially generalizes to the expected payoff conditional on a history in games with almost perfect information. The only difficulty is the concept of the node following an action $\nu(n, f)$, which we use in the reciprocation term. The problem is that there may be several nodes following a particular action since the next history is reached by an action combination. Let us use the notation of the previous section (Games with More That Two Players). Let h be a history in the game, let f be an end history in the game. We define $a_i(n, f)$ as the action leading in history h to the final history f. Since the links from one history to the next history are described by the action combinations of all players, $\nu_i(n, f)$ is well defined. If a_i is an action of player i in history h, then $\pi_j(n, a_i, s_i, s_j, s_{-ij})$ is defined as player j's expected payoff conditional on history h and action a_i, which is the expected payoff of player j in the subgame starting at history h, given a_i is played by player i in history h and all other strategies after h are given by the strategies s_i, s_j, and s_{-ij}. The reciprocation term is then given by $\pi_j(h, \nu_i(n, f), s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))}) - \pi_j(h, s_i^{(ji)}, s_i^{(j)}, s_i^{(j(-ij))})$.

8 Appendix 3: Propositions and Proofs

All predictions based on the more complex intention term presented in Appendix 1. Except for the mini best-shot game, the predictions do not differ from the predictions of a model with the easier intention term presented in section 2.

8.1 Ultimatum Game

In the ultimatum game, the first mover ("proposer") is allocated an amount of money (which we normalize to 1). The proposer has to divide this amount between himself and a second mover ("responder"). He may offer any feasible amount c to the responder, i.e., $0 \leq c \leq 1$. The offer is then revealed to the responder, who may either accept or reject it. If she accepts, the resulting payoffs are $1-c$ for the proposer and c for the responder. If the responder rejects the offer, payoffs are zero for both parties.

Given the standard assumptions, the outcome according to the subgame perfect Nash equilibrium is ($c = 0$; accept).

Proposition 3 *If ρ_1 and ρ_2 are positive there is a unique reciprocity equilibrium (c^*, p^*) in the ultimatum game as follows:*

$$p^* = \begin{cases} \min\left(1, \left[\frac{c}{\rho_2 \cdot (1-2c)(1-c)}\right]\right) & if\ c < \frac{1}{2} \\ 1 & if\ c \geq \frac{1}{2} \end{cases} \tag{17}$$

$$c^* = \max\left[\frac{1 + 3\rho_2 - \sqrt{1 + 6\rho_2 + \rho_2^2}}{4\rho_2}, \frac{1}{2} \cdot (1 - \frac{1}{\rho_1})\right] \tag{18}$$

If either ρ_1 or ρ_2 is zero p^* and c^* are the limits of the above formulas where ρ_1 and ρ_2 approach zero from above.

If ρ_1 and ρ_2 are both zero, $p^* = 1$ and $c^* = 0$.

Proof: Let p denote the probability that the responder accepts the offer; p' is the proposer's belief about p and p'' the responder's belief about p'. Let $\vartheta_1(c)$ be the intention factor at the node after player 1's choice of c. The responder's utility in the reciprocity game is given by U_{2A} in case she accepts the offer and by U_{2R} if she rejects the offer:

$$U_{2A} = c + \rho_2 \vartheta_1(c)(p''(c - (1 - c))) \cdot ((1 - c) - p''(1 - c))$$

$$U_{2R} = 0 + \rho_2 \vartheta_1(c)(p''(c - (1 - c))) \cdot (0 - p''(1 - c))$$

The difference in the responder's utility between accepting and rejecting amounts to:

$$U_{2A} - U_{2R} = c + \rho_2 \vartheta_1(c) p''(2c - 1)(1 - c) \tag{19}$$

For $c \geq \frac{1}{2}$, the expression $U_{2A} - U_{2R}$ is positive. Thus, offers of at least half of the pie are always accepted, i.e., $p = 1$. This proves the second part of equation (17) of the proposition.

Let us now turn to the case where $c < \frac{1}{2}$. In this case, the second summand in equation (19) is negative. We set equation (19) equal to zero, solve it for p'', and define this solution as p''_{crit}. We get:

$$p''_{crit} = \frac{c}{\rho_2 \vartheta_1(c)(1 - 2c)(1 - c)} \tag{20}$$

If $p'' < p''_{crit}$ then $U_{2A} > U_{2R}$. This implies that player 2 accepts, i.e., $p = 1$. This is an equilibrium if and only if $1 = p = p'' < p''_{crit}$. If, on the other hand, $p'' > p''_{crit}$ then $U_{2A} < U_{2R}$. This implies that player 2 rejects, i.e., $p = 0$. This, however, is impossible because it would imply that $0 = p = p'' > p''_{crit} > 0$. (The last inequality holds because $c < \frac{1}{2}$.) If, finally, $p'' = p''_{crit}$, then $U_{2A} = U_{2R}$. Therefore player 2 is indifferent between accepting and rejecting in this case and we get an equilibrium

$p = p'' = p''_{crit}$. This mixed equilibrium is possible for $0 \leq p''_{crit} \leq 1$. Taken together, we have shown that $p^* = \min\left(1, \frac{c}{\rho_2 \vartheta_1(c)(1-2c)(1-c)}\right)$.

Let us now discuss the role of intentions. If $c < \frac{1}{2}$ then player 2 is in the disadvantageous situation. Therefore, player 1's move is considered to be fully intentional if and only if player 1 has a possible action that leads to a higher payoff for player 2 without bringing player 1 into the disadvantageous situation. A contribution of $c = \frac{1}{2}$ satisfies this condition. Therefore, $\vartheta_1(c) = 1$ which completes the proof of equation (17).

Equation (17) yields the acceptance probability as a function of c. Let us denote this function as $p(c)$. Furthermore, we define $c_0 := \frac{1+3\rho_2-\sqrt{1+6\rho_2+\rho_2^2}}{4\rho_2}$. It is the smallest contribution c where p equals 1. We now turn to the proposer's behavior. His utility amounts to:

$$U_1 = p(c) \cdot (1-c) + \rho_1 \vartheta_2 \cdot p(c'') \cdot (1-2c'') \cdot (p(c)c - p(c'')c'') \tag{21}$$

In Equation (21) we already applied the consistency condition for p ($p = p' = p''$) in order to avoid confusion between p' as belief and the derivation of p as a function of c. We first note that Equation (21) is decreasing in c for $c'' \geq \frac{1}{2}$. Therefore, we can conclude that $c \leq \frac{1}{2}$. In this case, player 1 is in the advantageous situation and player 2's move is fully intentional if she has any alternative action that leads to a smaller payoff for player 1. For $c > 0$, this is always the case because player 2 can decrease player 1's payoff with respect to player 1's expectations by rejecting the offer. As we will see below, player 1's contribution is positive if both players have a positive reciprocity parameter. Therefore, we can assume $\vartheta_2 = 1$.

By using (20), it can be shown from equation (21) that the proposer's utility is increasing in c as long as his offer is strictly smaller than c_0 (and therefore the acceptance probability less than one). Thus, the optimal offer c^* of the responder is at least as high as c_0, i.e., $c^* \geq c_0$.

We now know that the proposer will choose c^* to be at least high enough to guarantee an acceptance probability of one. Therefore, we consider the situation where the proposer chooses an offer above c_0 (which is always accepted). If $p = 1$, we get:

$$U_{1(p=1)} = (1-c) + \rho_1 \cdot (1-2c'') \cdot (c - c'')$$

$$\frac{\partial U_{1(p=1)}}{\partial c} = -1 + \rho_1 \cdot (1-2c'')$$

We define

$$c''_{crit} = \frac{1}{2}\left(1 - \frac{1}{\rho_1}\right)$$

The utility U_1 is decreasing in c for $c'' > c''_{crit}$, U_1 is increasing in c for $c'' < c''_{crit}$, and U_1 is constant in c for $c'' = c''_{crit}$. Consider the case $c''_{crit} < c_0$: Since in equilibrium $c = c''$, we get $c''_{crit} < c_0 \leq c = c''$. Therefore, U_1 is decreasing in c and $c^* = c_0$ ($= \max(c_0, c''_{crit})$). Consider now the case $c''_{crit} \geq c_0$: If $c'' > c''_{crit}$, U_1 is decreasing in c and therefore c is chosen equal to c_0 which is incompatible with $c = c''$ because $c'' > c''_{crit} \geq c_0 = c$. If $c'' < c''_{crit}$, U_1 is increasing in c and therefore c is chosen equal to 1 which is also incompatible with $c = c''$ because $c'' < c''_{crit} < \frac{1}{2} < 1 = c$. Therefore $c^* = c'' = c''_{crit}$ $(= \max(c_0, c''_{crit}))$.

This completes the proof of the Proposition. ■

8.2 Random Move Ultimatum Game

In the random move ultimatum game, first, a random device determines the proposer's "offer". Then, the responder decides whether to accept or to reject the offer. Payoffs are the same as in the ultimatum game.

Proposition 4 *In the random move ultimatum game there is a unique reciprocity equilibrium* (p^*) *as follows:*

$$p^* = \begin{cases} \min\left(1, \left[\frac{c}{\varepsilon_2\rho_2\cdot(1-2c)(1-c)}\right]\right) & \textit{if } c < \frac{1}{2} \\ 1 & \textit{if } c \geq \frac{1}{2} \end{cases}$$

Proof: Since the first mover has no choice at all, the intention factor ϑ_1 at the decision node of the responder equals ε_2 instead of 1. The responder in this game has the same utility as a responder in the ordinary ultimatum game whose reciprocity parameter equals $\varepsilon_2\rho_2$. Hence, using the solution for the standard ultimatum game, we get the proposition. ■

8.3 Gift-Exchange Game

The gift-exchange game is a two-person sequential move game. In this game, the first mover (called an employer) offers a wage w to a second mover (called a worker). Then, the worker has to make an effort decision e. Providing effort above the minimum effort level is costly with $c(e)$ being a convex effort cost function. Payoff functions are given by $\pi_1 = ve - w$ for employers and $\pi_2 = w - c(e)$ for workers, respectively. In our analysis we restrict $w \in [0,1]$, $e \in [0,1]$, we normalize v to 1 and write the convex effort cost function as $c(e) = \alpha e^2$ with $\alpha \leq \frac{1}{4}$. To ease the notation in the next proposition, we use the following definitions. First, let $\widetilde{w}(\alpha, \rho_1, \rho_2)$ be the wage that would be chosen by the employer if w and e would not be restricted to be smaller than 1. The exact formula for $\widetilde{w}(\alpha, \rho_1, \rho_2)$ is given in the proof of the proposition. We further define $\overline{w}(\alpha, \rho_2) = \frac{1+\alpha}{2} + \frac{\alpha}{\rho_2}$ which is the minimal wage that guarantees an effort choice of one.

Proposition 5 *In the gift-exchange game there are the following reciprocity equilibria* (w^*, e^*)*:*

(i) If $\rho_2 = 0$*, then there exists a unique equilibrium:*

$$w^* = e^* = 0 \tag{22}$$

(ii) If $\rho_2 > 0$*, then*

$$e^* = \min\left(1, \frac{-2\alpha - \rho_2 + \sqrt{(2\alpha+\rho_2)^2 + 8\alpha\rho_2^2 w}}{2\alpha\rho_2}\right) \tag{23}$$

There is always an equilibrium given by:

$$w^* = \min\left(1, \max\left(0, \min\left(\overline{w}(\alpha,\rho_2), \widetilde{w}(\alpha,\rho_1,\rho_2)\right)\right)\right) \tag{24}$$

If $\varepsilon_1\rho_1 \leq \frac{\rho_2(-\rho_2+2\alpha+2\alpha\rho_2)}{2\alpha(-2\alpha-\rho_2+2\alpha\rho_2)}$ *and* $\overline{w}(\alpha,\rho_2) \leq 1$, *then there is a second equilibrium which is given by:*

$$w^* = \overline{w}(\alpha,\rho_2)$$

Proof:

If $\rho_2 = 0$, the worker chooses $e = 0$ since her material payoff decreases in e. Given this, the kindness of player 2 is smaller or equal to zero. Therefore, an increase in w reduces player 1's material payoff and his reciprocity utility. Hence, $w^* = e^* = 0$ is the unique equilibrium for $\rho_2 = 0$.

Let us now assume that $\rho_2 > 0$.

For $w = 0$, the worker chooses an effort of zero because her material payoff and her reciprocity utility decrease in e in this case. If $w > 0$, the worker always chooses an effort such that her payoff is strictly greater than the employer's payoff. If the worker chooses $e^* = 0$, then this is obviously the case. If this would not be the case for $e^* > 0$, the worker could increase her utility by reducing her effort. This would increase her own payoff and decrease that of the employer, both leading to an increase in her utility. Consequently, the employer is always kind to the worker and the worker is always unkind to the employer.

The worker's utility equals:

$$U_2 = w - \alpha e^2 + \rho_2\vartheta_1(w)((w - \alpha e''^2) - (e'' - w))(e - w - (e'' - w))$$

We differentiate U_2 with respect to e, then set $e = e''$ and we get

$$e^* = \min\left(1, \frac{-(2\alpha + \rho_2\vartheta_1(w)) + \sqrt{(2\alpha + \rho_2\vartheta_1(w))^2 + 8\alpha(\rho_2\vartheta_1(w))^2 w}}{2\alpha\rho_2\vartheta_1(w)}\right)$$

We see that the worker provides a strictly positive effort for a strictly positive wage. Now, we show that if the employer pays a positive wage, this is fully intentional which implies $\vartheta_1(w) = 1$: We have seen that the worker provides an effort that results in a higher payoff for the worker than for the firm, i.e., the worker will always get at least half of the cake size. The total cake size to distribute is $e - c(e)$ which is positive for all choices of e. Therefore, if $w > 0$, then $\pi_2(w, e(w)) > 0 = \pi_2(0,0)$, because the firm can reduce the worker's payoff by choosing 0, $\vartheta_1(w) = 1$ for $w > 0$. This concludes the proof of equation (23)

We now turn to the employer's behavior. As we have seen above, the worker is unkind if the employer pays a positive wage. This unkindness is fully intentional if and only if the effort provided is strictly smaller than 1. (If $e < 1$, the worker could - without switching into a disadvantageous situation - improve the employer's profit by choosing a slightly higher effort. This is impossible if $e = 1$.) Therefore, there are two possible reciprocity equilibria: (i) The employer chooses the minimal wage that guarantees an effort of 1. The worker indeed chooses an effort of 1. The unkindness of the worker is unintentional in this case. (ii) The employer pays a wage such that the worker provides an effort smaller than 1. The worker actually chooses her effort fully intentionally in this case.

Let us first consider the case in which the employer pays the minimal wage that guarantees an effort of 1. We have defined this wage as $\overline{w}$. To check under which conditions the employer chooses this strategy, we check whether a reduction of the wage would reduce the employer's utility. We calculate

$$\overline{w}(\alpha, \rho_2) = \frac{1}{2\rho_2}\left(2\alpha + \rho_2 + \alpha\rho_2\right)$$

$$\frac{\partial e}{\partial w} = \frac{2\rho_2}{\sqrt{\left(4\alpha^2 + 4\alpha\rho_2 + \rho_2^2 + 8\alpha\rho_2^2 w\right)}}$$

$$\frac{\partial e}{\partial w}(w = \overline{w}) = \frac{2\rho_2}{2\alpha + \rho_2 + 2\alpha\rho_2}$$

and get

$$U_1 = e(w) - w + \rho_1\varepsilon_1(e(w'') + \alpha e(w'') - 2w)(w - \alpha e(w)^2 - (w'' - \alpha e(w'')^2))$$

and

$$\frac{\partial U_1}{\partial w} = \frac{\partial e(w)}{\partial w} - 1 + \rho_1\varepsilon_1(e(w'') + \alpha e(w'') - 2w)(1 - 2\alpha e(w)\frac{\partial e(w)}{\partial w})$$

Now we set $\frac{\partial U_1}{\partial w}(\overline{w}) = 0$ and get a critical

$$(\rho_1\varepsilon_1)_{crit} = \frac{\rho_2\left(-\rho_2 + 2\alpha + 2\alpha\rho_2\right)}{2\alpha\left(-2\alpha - \rho_2 + 2\alpha\rho_2\right)}$$

If the actual value of $\rho_1\varepsilon_1$ is below this critical value, $\overline{w}$ is a local optimum of the employer's utility. This local maximum is global since e is a concave function in w.

This establishes the existence of the first type of equilibrium.

It now remains to consider The case where the workers behave fully intentionally must also be considered, i.e., $e(w^*) < 0$ and $\vartheta_2 = 1$. The calculation of this equilibrium leads to $\widetilde{w}(\alpha, \rho_1, \rho_2)$, with

$$\widetilde{w}(\alpha, \rho_1, \rho_2) = \begin{cases} \frac{-4\alpha^2 - 4\alpha\rho_2 + 3\rho_2^2}{8\alpha\rho_2^2} & \text{if } \rho_1 = 0 \\ \frac{3}{16\alpha} + \frac{3}{4\rho_1} + \frac{3\alpha}{4\rho_2^2} + \frac{3}{4\rho_2} + \frac{\rho_2}{8\alpha\rho_1} + \frac{\rho_2^2}{16\alpha\rho_1^2} - \\ -\frac{(6\alpha\rho_1 + 3\rho_1\rho_2 + \rho_2^2)\sqrt{4\alpha^2\rho_1^2 + 4\alpha\rho_1^2\rho_2 + 12\alpha\rho_1\rho_2^2 + \rho_1^2\rho_2^2 - 2\rho_1\rho_2^3 + \rho_2^4}}{16\alpha\rho_1^2\rho_2^2} & \text{if } \rho_1 > 0 \end{cases}$$

If $\widetilde{w}(\alpha, \rho_1, \rho_2) \leq \overline{w}(\alpha, \rho_2)$, we get $w^* = \widetilde{w}(\alpha, \rho_1, \rho_2)$. If $\widetilde{w}(\alpha, \rho_1, \rho_2) > \overline{w}(\alpha, \rho_2)$ the employer chooses $w^* = \overline{w}(\alpha, \rho_2)$ because for wages above $\overline{w}(\alpha, \rho_2)$ the employer's material payoff as well as the reciprocity utility are decreasing in w. This concludes the proof. ■

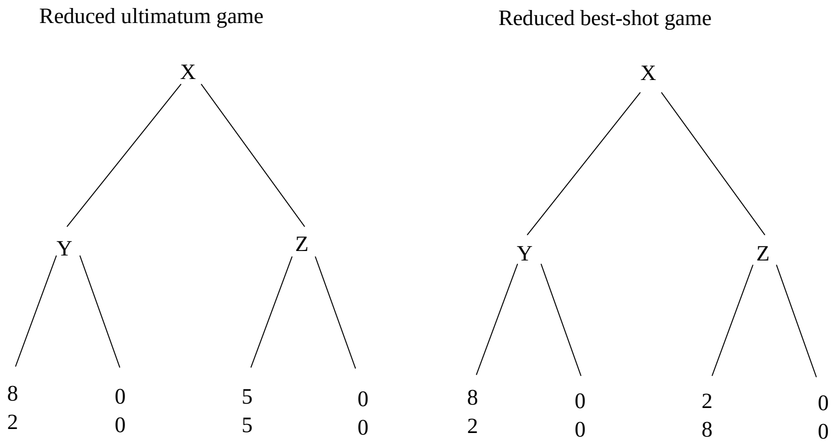

Figure 3: The game trees of a reduced ultimatum game and a reduced best-shot game.

8.4 Reduced Ultimatum Game, Reduced Best-Shot Game

The reduced ultimatum game and the reduce best-shot game are given by the game trees in Figure 3.

Let p be the probability of the proposer choosing (8,2). Let q be the acceptance probability of the (8,2) offer and r the acceptance probability of the alternative. The games are normalized with a factor of $\frac{1}{10}$.

Proposition 6 *(Ultimatum game) In the normalized reduced ultimatum game there is a unique reciprocity equilibrium. It is given by:*

$$r^* = 1$$

$$q^* = \min\left(1, \frac{5}{12}\frac{1}{\rho_2}\right) \tag{25}$$

$$p^* = \min\left(1, \max\left(0, \frac{0.8q^* - 0.5}{0.6(0.5 - 0.2q^*)q^*\rho_1}\right)\right) \tag{26}$$

(Best-shot game) In the normalized reduced best-shot game the reciprocity equilibria are described as follows:

$$r^* = 1$$

$$q^* = \min\left(1, \frac{5}{12}\frac{1}{\varepsilon_2\rho_2}\right) \tag{27}$$

If $\rho_2\varepsilon_2 < \frac{5}{3}$

$$p^* = \min\left(1, \max\left(0, \frac{1+\frac{0.8q^*-0.2}{0.6(0.8-0.2q^*)\rho_1}}{1+q^*}\right)\right) \tag{28}$$

If $\rho_2\varepsilon_2 = \frac{5}{3}$ *and* $\rho_1 = 0$ *then any* $p^* \in [0,1]$ *is part of an equilibrium. If* $\rho_2\varepsilon_2 = \frac{5}{3}$ *and* $\rho_1 > 0$ *then* $p^* = \frac{4}{5}$.

If $\rho_2\varepsilon_2 > \frac{5}{3}$ *then*

$$p^* = \begin{cases} min\left(1, max\left(0, \frac{1+\frac{0.8q^*-0.2}{0.6(0.8-0.2q^*)\rho_1}}{1+q^*}\right)\right) \\ \quad if\ \rho_1 \leq \frac{10-40q^*}{12-15q^*+3q^{*2}} \\ min\left(1, max\left(0, \frac{2+\frac{0.8q^*-0.2}{0.6(0.8-0.2q^*)\rho_1}}{3+q^*}\right)\right) \\ \quad if\ \frac{10-40q^*}{12-15q^*+3q^{*2}} \leq \rho_1 \leq \frac{5(3-11q^*-4q^{*2}+\varepsilon_1(-1+3q^*+4q^{*2}))}{3\varepsilon_1(4-5q^*+q^{*2})} \\ min\left(1, max\left(0, \frac{1+\frac{0.8q^*-0.2}{0.6(0.8-0.2q^*)\varepsilon_1\rho_1}}{1+q^*}\right)\right) \\ \quad if\ \frac{5(3-11q^*-4q^{*2}+\varepsilon_1(-1+3q^*+4q^{*2}))}{3\varepsilon_1(4-5q^*+q^{*2})} \leq \rho_1 \end{cases} \tag{29}$$

Proof: Let us first consider the situation of player 2 in which she has to decide whether to accept or reject some distribution with a_1 for player 1 and a_2 for player 2. Let U_A denote the utility of player 2 if she accepts and U_R the utility of player 2 if she rejects the offer. Let t be the acceptance probability and n the node where player 2 has to decide. We get

$$U_A = a_2 + \rho_2\vartheta_1(n)(t''(a_2 - a_1))(a_1 - \pi_1(n, s_2'', s_2'))$$

$$U_R = 0 + \rho_2\vartheta_1(n)(t''(a_2 - a_1))(0 - \pi_1(n, s_2'', s_2'))$$

and

$$U_A - U_R = a_2 + \rho_2\vartheta_1(n)(t''(a_2 - a_1))a_1$$

We immediately see that the offer is always accepted if $a_2 \geq a_1$. Therefore, $r^* = 1$ in both reduced games.

First, q^*, is derived analogously to the proof of the ultimatum game equilibrium:

$$q^* = \min\left(1, \frac{0.2}{0.8\rho_2\vartheta_1(Y)(0.8-0.2)}\right) = \min\left(1, \frac{5}{12\rho_2\vartheta_1(Y)}\right)$$

We now have to calculate the value of $\vartheta_1(Y)$, the intention factor (Y corresponds to node Y, see Figure 1). In the reduced ultimatum game it is equal to 1 because the proposer has the possibility to offer $(0.5, 0.5)$ which results in a higher payoff for 2 compared to $(0.8, 0.2)$ $(0.5 > q'0.2)$ and which is reasonable because the proposer still receives at least as much as the responder. This proves (25).

In the reduced best-shot game, the proposer's alternative strategy$(0.2, 0.8)$ puts himself into a disadvantageous situation. This disadvantageous inequity is at least as large as the advantageous inequity of the reference choice of $(0.8, 0.2)$ (it is 0.6 compared to $0.6q'$). The alternative $(0.2, 0.8)$ therefore has an intention factor of ε_2. Hence, in this case, $\vartheta_1(Y) = \varepsilon_2$ which proves (27).

In the reduced ultimatum game the second mover is always kind and could be less kind by rejecting the offer. Therefore, player 1's optimal strategy can be calculated with an intention factor of 1. We get

$$U_{82} = 0.8q + \rho_1(0.8p''q + 0.5(1-p'') - (0.2p''q + 0.5(1-p'')))(0.2q - \pi_2(n, s_1'', s_1'))$$

$$U_{55} = 0.5 + \rho_1(0.8p''q + 0.5(1-p'') - (0.2p''q + 0.5(1-p'')))(0.5 - \pi_2(n, s_1'', s_1'))$$

$$U_{82} - U_{55} = (0.8q - 0.5) + \rho_1(0.8 - 0.2)p''q(0.2q - 0.5)$$

To get (26), we set $U_{82} - U_{55} = 0$ and apply the same arguments as in the proof of ultimatum game proposition. This proves equation (26).

In the reduced best-shot game, the situation is somewhat more complicated. We get

$$U_{82} = 0.8q + \rho_1\vartheta_2(X)(0.8p''q + 0.2(1-p'') - (0.2p''q + 0.8(1-p'')))(0.2q - \pi_2'')$$

$$U_{28} = 0.2 + \rho_1\vartheta_2(X)(0.8p''q + 0.2(1-p'') - (0.2p''q + 0.8(1-p'')))(0.8 - \pi_2'')$$

$$U_{82} - U_{28} = (0.8q - 0.2) + \rho_1\vartheta_2(X)((0.8 - 0.2)(p''q - (1-p'')))(0.2q - 0.8)$$

and therefore the equilibrium acceptance probability p^* equals:

$$p^* = \min\left(1, \max\left(0, \frac{1 + \frac{0.8q^* - 0.2}{0.6(0.8 - 0.2q^*)\vartheta_2(X)\rho_1}}{1 + q^*}\right)\right)$$

For the value of $\vartheta_2(X)$, we have to consider the intention of player 2. First, we show that in equilibrium $\Delta_2(X)$ is positive if and only if $0.8q^* - 0.2$ is positive: We get $\Delta_2(X) = 0.6(p^*q^*) - 0.6(1 - p^*)$. If $p^* \in (0, 1)$, then $\Delta_2(X)$ is positive if and only if $0 < p^*q^* - (1 - p^*) = p^*(q^* + 1) - 1 = \frac{0.8q^* - 0.2}{0.6(0.8 - 0.2q^*)\rho_1}$. Therefore, in this case, $\Delta_2(X)$ is positive if and only if $0.8q^* - 0.2$ is positive. If $p^* = 1$ then $\Delta_2(X) = 0.6(p^*q^* - (1 - p^*)) = 0.6q^*$, which implies $\Delta_2(X) > 0$. Note that $p^* = 1$ can only occur if $0.8q^* - 0.2 > 0$. If $p^* = 0$ then $\Delta_2(X) = 0.6(p^*q^* - (1 - p^*)) = -0.6$, therefore $\Delta_2(X) < 0$. This can only be the case if $0.8q^* - 0.2 < 0$.

If $0.8q^* - 0.2 > 0$, i.e., $q^* > \frac{1}{4}$, player 2 is kind and can be less kind by rejecting any offer. Therefore, in this case, $\vartheta_2(X) = 1$. This proves (28).

If $q^* = \frac{1}{4}$, then player 1 is materially indifferent between his two possible actions. If $\rho_1 = 0$, he is also indifferent with respect to his reciprocity utility. If $\rho_1 > 0$, then only $p^* = \frac{4}{5}$ can be part of an equilibrium: This choice leads to an equitable expected payoff of $(0.2, 0.2)$. Any other choice of p^* would lead to some inequity. Because player 1 is materially indifferent between the two choices, he would reciprocate this inequity and try to produce an inequity in the opposite direction. Therefore, $p^* = \frac{4}{5}$ is the only equilibrium choice in this case.

If $0.8q^* - 0.2 < 0$, i.e., $q^* < \frac{1}{4}$, player 2 is unkind but could be less unkind if she would accept both offers, i.e., if she would choose $q = 1$. This is reasonable if $p \leq \frac{1}{2}$ because in this case, player 2 is still in the advantageous position. The inequality $p \leq \frac{1}{2}$ is satisfied if $\rho_1 \leq \frac{10-40q^*}{12-15q^*+3q^{*2}}$. If $p > \frac{1}{2}$, the intention factor is

$$\vartheta_2(X) = \max\left(\varepsilon_1, 1 - \frac{-0.6p'' + 0.6(1-p'')}{0.6p''q' - 0.6(1-p'')}\right) \tag{30}$$

If the first term applies, we get

$$p^* = \min\left(1, \max\left(0, \frac{1 + \frac{0.8q^* - 0.2}{0.6(0.8-0.2q^*)\varepsilon_1\rho_1}}{1+q^*}\right)\right)$$

The first term applies if

$$1 - \frac{-0.6p^* + 0.6(1-p^*)}{0.6p^*q^* - 0.6(1-p^*)} \leq \varepsilon_1$$

which is the case if

$$\rho_1 \geq \frac{5(3 - 11q^* - 4q^{*2} + \varepsilon_1(-1 + 3q^* + 4q^{*2}))}{3\varepsilon_1\left(4 - 5q^* + q^{*2}\right)}$$

Finally, if $\rho_1 < \frac{5(3-11q^*-4q^{*2}+\varepsilon_1(-1+3q^*+4q^{*2}))}{3\varepsilon_1(4-5q^*+q^{*2})}$ then in (30), the second term applies and

$$p^* = \min\left(1, \max\left(0, \frac{2 - \frac{(0.2-.8q^*)}{0.6(0.8-0.2q^*)\rho_1}}{3+q^*}\right)\right)$$

which concludes the proof. ■

8.5 Proposer Competition Game

In the proposer competition game, there are $n-1$ proposers who simultaneously propose an offer $c_i \in [0,1]$ to the responder with $i \in \{1, ..., n-1\}$. These offers are revealed to the responder who has to decide whether to accept or reject the highest offer c_{max}. If more than one proposer offers c_{max} a random mechanism determines whose offer will be selected. Payoffs are exactly as in the ultimatum game, i.e., the proposer whose offer is accepted receives $1 - c_{max}$ and the responder gets c_{max}. A proposer whose offer is not accepted receives a payoff of zero. If the responder rejects c_{max}, all receive nothing.

Proposition 7 *In a reciprocity equilibrium in the proposer competition game, at least 2 proposers offer* $c_{max} = 1$ *which the responder accepts.*

Proof:

First note that the responder will accept any offer above 0.5. This holds because accepting yields a higher material payoff than rejecting. Moreover, accepting provides a higher reciprocity utility with respect to the proposer who offers $c_{\max}$ because this proposer is kind towards the responder. Furthermore, the responder's reciprocity utility with respect to the other proposers equals zero, independent of the responder's decision because the reciprocation term equals zero in this case.

If at least two players offer $c = 1$, then any offer is a best response for all proposers since all payoffs are unaffected by their decisions. Therefore, the strategies described in the Proposition are indeed an equilibrium. Only one proposer offering $c = 1$ cannot be an equilibrium since this player has an incentive to lower his offer slightly: this increases his material payoff and the reciprocity utility towards the responder. Changing the offer from $c = 1$ has no effect on the reciprocity utility towards the other responders because $\Delta = 0$. If $c_{\max} < 1$, then all proposers have an incentive to increase their offer to $c_{\max}$. If at least two proposers offer $c_{\max} < 1$ then these proposers have an incentive to increase their offer infinitesimally. This increases the material payoff from $\frac{c_{\max}}{k}$ to $c_{\max}$, where k is the number of proposers offering $c_{\max}$. On the other hand, it changes the reciprocity utilities only infinitesimally. Hence $c_{\max} < 1$ cannot be an equilibrium. ■

8.6 Dictator Game

The dictator game is a very simple two person game. The task of the first mover (the so-called "dictator") is to divide an amount of money between himself and a counterpart (the "receiver"). Let 1 be the amount of money and c the share for the receiver. The dictator is free to choose any feasible division he wants ($0 \leq c \leq 1$). The receiver has no choice to make, i.e., she has to accept any amount sent to her. The payoff for the receiver is simply the amount c she has been sent by the dictator. The dictator's payoff is given by the residual amount $1 - c$.

Proposition 8 *In the dictator game, there is a unique reciprocity equilibrium. It is given by*

$$c^* = \max\left(0, \frac{1}{2} \cdot \left(1 - \frac{1}{\varepsilon_1 \rho_1}\right)\right)$$

Proof: Since the receiver has no choice, the outcome is not intentional and therefore the intention factor at the decision node of player 1 equals ε_1. Therefore

$$U_1 = (1 - c) + \rho_1 \varepsilon_1 (1 - c'' - c'')c$$

The first order condition determines a $c_{crit} = \frac{1}{2}\left(1 - \frac{1}{\varepsilon_1 \rho_1}\right)$. The rest of the proof follows as in the proof of the ultimatum game. ■

8.7 Sequential Prisoner's Dilemma Game

The sequential prisoner's dilemma consists of two stages. At the first stage, player 1 can either cooperate or defect. After observing player 1's choice, player 2 has the same choice, i.e., to either cooperate or to defect. The resulting payoffs are a_3 for players 1 and 2 if both cooperate, a_2 for players 1 and 2 if both defect and a_1 for the cooperator and a_4 for the defector, if one of players 1 and 2 cooperates and the other one defects. In the prisoner's dilemma the following inequalities hold:

$$a_1 < a_2 < a_3 < a_4 \tag{31}$$

The subgame perfect outcome of the standard game is that both players defect.

Let p denote the probability that player 1 cooperates. Let q denote player 2's cooperation probability if player 1 cooperates and r be the probability that player 2 cooperates if player 1 does not cooperate. Let X be player 1's decision node, let Y be player 2's decision node after player 1 cooperated and Z player 2's decision node after player 1 defected.

Proposition 9 *In the sequential prisoner's dilemma game there is a unique reciprocity equilibrium. If $\rho_1, \rho_2 > 0$ it is given by:*

$$r^* = 0 \tag{32}$$

$$q^* = \max\left(0, 1 - \frac{a_4 - a_3}{\rho_2(a_3 - a_1)(a_4 - a_1)}\right) \tag{33}$$

$$p^* = \max\left(0, \min\left(1, \frac{q^* a_3 + (1 - q^*)a_1 - a_2}{\rho_1(a_4 - a_1)(1 - q^*)(q^* a_3 + (1 - q^*)a_4 - a_2)}\right)\right) \tag{34}$$

If ρ_1 or ρ_2 are zero, p^* and q^* are the limits of the above formulas where ρ_1 and ρ_2 approach zero from above.

Proof: First, we prove Equation (32). Let U_{2DC} and U_{2DD} denote player 2's utility if player 1 defected and if player 2 cooperates or defects:

$$U_{2DC} = a_1 + \rho_2 \vartheta_1(Z)(r''(a_1 - a_4) + (1 - r'')(a_2 - a_2))(a_4 - \pi_1(Z, s_2'', s_2'))$$

$$U_{2DD} = a_2 + \rho_2 \vartheta_1(Z)(r''(a_1 - a_4) + (1 - r'')(a_2 - a_2))(a_2 - \pi_1(Z, s_2'', s_2'))$$

Because the kindness term in both equations is negative and $a_2 > a_1$ and $a_2 < a_4$, defection strictly dominates cooperation independent of the value of $\vartheta_1(Z)$. This proves equation (32).

Next, we prove equation (33). Let us first consider player 1's intention factor at node Y. Since the outcome term $q''(a_3 - a_3) + (1 - q'')(a_4 - a_1)$ is positive, player 2 is treated kindly. Hence, it is sufficient to show that player 1 has a move that leads to a smaller payoff for player 2. Indeed, if player 1 cooperates, player 2 always gets a payoff between a_3 and a_4. This is more than if player 1 defects - in which case player 2 gets only a_2. Therefore, player 1's action is fully intentional, i.e., $\vartheta_1(Y) = 1$. Let U_{2CC} and U_{2CD} denote player 2's utility if player 1 cooperates and player 2 either cooperates or to defects.

$$U_{2CC} = a_3 + \rho_2(q''(a_3 - a_3) + (1 - q'')(a_4 - a_1))(a_3 - \pi_1(Y, s_2'', s_2'))$$

$$U_{2CD} = a_4 + \rho_2(q''(a_3 - a_3) + (1 - q'')(a_4 - a_1))(a_1 - \pi_1(Y, s_2'', s_2'))$$

Player 2 is indifferent between cooperation and defection if

$$U_{2CC} - U_{2CD} = 0 \tag{35}$$

holds. Let us define q''_{crit} as the second order belief q'' which solves Equation (35). We get the following expression:

$$q''_{crit} := 1 - \frac{a_4 - a_3}{\rho_2(a_3 - a_1)(a_4 - a_1)} \tag{36}$$

The rest of the proof of Equation (33) is analogous to the proof of the ultimatum game equilibrium.

To prove (34), we calculate:

$$\begin{aligned} \Delta_2(X) &= p''(q'a_3 + (1 - q')a_1) + (1 - p'')(r'a_4 + (1 - r')a_2) - \\ &\quad -(p''(q'a_3 + (1 - q')a_4) + (1 - p'')(r'a_1 + (1 - r')a_2)) \end{aligned}$$

Applying the consistency conditions ($q' = q^*$ and $r' = r^* = 0$) we get the expression:

$$\Delta_2(X) = p''(1 - q^*)(a_1 - a_4)$$

Player 2 is always in the advantageous situation because $q^* < 1$. Therefore, player 2 is unkind. To evaluate the intention factor of player 2, we have to find an alternative strategy of player 2 that leads to a higher payoff for player 1 and does not put player 2 into a disadvantageous situation. The fully conditional cooperation ($q = 1$ and $r = 0$) is a strategy that satisfies this condition for $p'' > 0$. Therefore, if $p'' > 0$, player 2's behavior is fully intentional and $\vartheta_2(X) = 1$. For $p'' = 0$, the kindness term is equal to 0, and so $\vartheta_2(X)$ does not matter. For the analysis of player 1's strategy, we can therefore assume that $\vartheta_2(X) = 1$.

Let U_{1C} be the expected utility for player 1 if he cooperated and let U_{1D} be the expected utility for player 1 if he defects. We get

$$U_{XC} = qa_3 + (1 - q)a_1 + \rho_1\Delta_2(X)(qa_3 + (1 - q)a_4 - \pi_2''(X))$$

$$U_{XD} = ra_4 + (1 - r)a_2 + \rho_1\Delta_2(X)(ra_1 + (1 - r)a_2 - \pi_2''(X))$$

Using these formulas, setting $U_{XC} - U_{XD} = 0$ and solving for p'' we get a critical p'' , which we will call p''_{crit}:

$$p''_{crit} = \frac{q^* a_3 + (1 - q^*)a_1 - a_2}{\rho_1(a_4 - a_1)(1 - q^*)(q^* a_3 + (1 - q^*)a_4 - a_2)}$$

Again, the rest of the proof of Equation (34) is analogous to the proof of the ultimatum game equilibrium. ■

8.8 Simultaneous Prisoner's Dilemma Game

Proposition 10 *In the simultaneous prisoner's dilemma game there is no cooperation, i.e., the reciprocity equilibrium is given by* $p^* = q^* = 0$.

Proof : Suppose the first player cooperates with probability p_1, the second with probability p_2. Let p'_1, p'_2, p''_1, p''_2 be the corresponding beliefs of first and second order. As long as $p_1 > p_2$ holds, player 1 can increase his material payoff *and* his reciprocity utility by reducing his cooperation probability. Therefore, in equilibrium, $p_1 = p_2$ holds. If, however, $p_1 = p_2$ holds, then the second order beliefs are also equal, i.e., $p''_1 = p''_2$. This means that player 1's kindness term is zero and, consequently, utility arising from his reciprocity motive is zero. Only his concern for the material payoff matters for his decision. Consequently, he can increase his utility by reducing his cooperation probability. Therefore, as long as $p_1 > 0$, player 1 increases his utility by lowering his cooperation probability. Since the same argument holds for player 2, the unique equilibrium is $p_1 = p_2 = 0$, i.e., defection of both players. This result is independent of the reciprocal parameters ρ_1 and ρ_2 and the outcome concern parameters ε_1 and ε_2. ■

8.9 Public Goods Games

In the public goods game we analyze how n players simultaneously decide about how much of an endowment of 1 they contribute to a public good. Their payoffs consist of the money they keep and of the value of the public good. We consider a linear public good for which the value is $\gamma \sum_j g_j$ with a marginal per capita return of an investment into the public good $\gamma \in (\frac{1}{n}, 1)$. Let g_j be player j's contribution to the public good. Then player i's payoff equals $\pi_i = 1 - g_i + \gamma \sum_j g_j$. Because $\gamma < 1$, contributing nothing is a dominant strategy. Because $\gamma > \frac{1}{n}$, contributing 1 would be efficient.

Proposition 11 *(i) In the public goods game, there is a unique reciprocity equilibrium with* $g_i^* = 0$ *for all* i.

(ii) In a two player sequential public goods game, the second mover will choose the cooperation level $g_2^* = \max\left(0, g_1 - \frac{1-\gamma}{\gamma\rho_2}\right)$

Proof: In the public goods game, we get

$$\pi_i - \pi_k = 1 - g_i + \gamma \sum_j g_j - (1 - g_k + \gamma \sum_j g_j) = g_k - g_i.$$

and

$$\frac{\delta U_i}{\delta g_i} = -1 + \gamma + \rho_i \sum_{k \neq i} \vartheta_k (g_k'' - g_i'')\gamma$$

As in the proofs above, we get a critical

$$g_i^{crit} = \frac{\sum_{k \neq i} \vartheta_k g_k}{\sum_{k \neq i} \vartheta_k} - \frac{1-\gamma}{\gamma \rho_i (n-1)} \tag{37}$$

and conclude that in equilibrium

$$g_i^* = \max \left(0, \frac{\sum_{k \neq i} \vartheta_k g_k}{\sum_{k \neq i} \vartheta_k} - \frac{1-\gamma}{\gamma \rho_i (n-1)} \right) \tag{38}$$

(This equation is valid for $\rho_i > 0$. For $\rho_i = 0$, it is the limit from above of this expression, i.e., $g_i^* = 0$.) The first term is a weighted average of the others' contributions. Hence, this equation means that player i is willing to invest $\frac{1-\gamma}{\gamma \rho_i (n-1)} (> 0)$ less than a weighted average of the other players. Let now h be the player with the highest contribution, i.e., $g_k^* \leq g_h^*$ for all k. We get $\frac{\sum_{k \neq i} \vartheta_k g_k^*}{\sum_{k \neq i} \vartheta_k} - \frac{1-\gamma}{\gamma \rho_i (n-1)} \leq g_h^* - \frac{1-\gamma}{\gamma \rho_i (n-1)} < g_h^*$. Therefore, $g_h^* = 0$ and since $g_k^* \leq g_h^*$, we get $g_k = 0$ for all k which proves (i).

We now come to the proof of (ii). Assume player 1 moves first and player 2 moves second. First, if $g_1 = 0$, also $g_2 = 0$. Now consider the case $g_1 > 0$. In this case, we get $g_2 < g_1$ and therefore $\pi_2 > \pi_1$. Hence, we are in the domain of positive reciprocity and have to check whether player 1 could have made a move with a lower payoff for player 2. This is indeed the case for $g_1 = 0$ since

$$\begin{aligned} \pi_2(g_1, g_2) &= 1 - g_2 + \gamma(g_1 + g_2) \geq \\ &\geq 1 - g_2 + \gamma(g_2 + g_2) > 1 = \pi_2(0, 0) \end{aligned}$$

where the last inequality holds since $\gamma > \frac{1}{2}$. Therefore, $\vartheta = 1$ which concludes the proof of (ii). ■

8.10 The Centipede Game

Consider the centipede game in Figure 4.

Proposition 12 *If* $\rho_2 > \frac{1}{57}$ *or* $\rho_1 > \frac{1}{24}$, *then there is a reciprocity equilibrium in which player 1 passes in A with a strictly positive probability.*

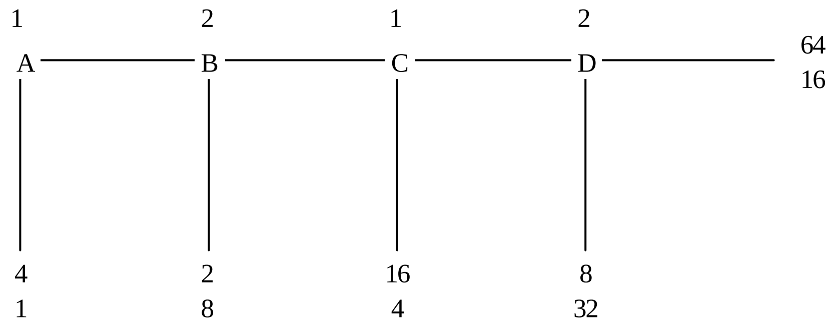

Figure 4: Game Tree of a centipede game

Proof: Let d_I be the probability that the player 'takes' at node $I \in \{A, B, C, D\}$. The utility as defined in the theory allows backward induction. To see this, consider decision node D. Utility for 'take' equals

$$U_t^D = 32 + \rho_2 \left[\varphi_B \sigma_B + \vartheta^D (d''_D(32-8) + (1-d''_D)(16-64))8\right]$$

where φ_B is the kindness as perceived in node B and σ_B is the reciprocation term in B which is the change in player 1's expected payoff when moving from node B to node C. With this notation, utility for passing equals

$$U_p^D = 16 + \rho_2 \left[\varphi_B \sigma_B + \vartheta^D (d''_D(32-8) + (1-d''_D)(16-64))64\right]$$

Player 2 chooses 'take' if $16 > \rho_2 \vartheta^D (d''_D 72 - 48)56$ or $d''_D < \frac{1}{72}(\frac{16}{56_{\rho_2}\vartheta^D} + 48) \equiv d_D^{crit}$. Note, that this formula is independent of beliefs about earlier moves. Further note that $\vartheta^D = 1$ because player 1 could have chosen 'take' at node A in which case player 2 would have earned 1, which is less than 32 or 16. So we define $d_D^{crit} = \frac{1}{72}(\frac{16}{56_{\rho_2}} + 48)$. If $d_D^{crit} \geq 1$, player 2 will take and for lower values, player 2 takes with probability d_D^{crit}.

Let us now turn to player 1's move in C. If $\rho_2 > \frac{1}{57}$, then player 2 takes with a probability lower than $\frac{50}{56}$. Therefore, it is materially profitable for player 1 to pass in C. Therefore, if player 1 is selfish, he passes. It can be shown as in the analysis of move D that a player 1 with low ρ_1 will pass with probability 1. A player 1 with a high reciprocity parameter will take with positive probability. However, independent of the reciprocity parameter of player 1, the expected payoff in C will be higher for player 2 than for player 1 which implies that player 1 passes with a strictly positive probability.

For player 2, passing in B is materially better than taking since by passing player 2 will get more than 50% of a cake of at least 20. Furthermore, since player 2 gets a higher expected payoff than 1 when passing, reciprocation can only induce player 2 to reward player 1. But to reward player 1, player 2 also has to pass.

Looking at player 1's move in A, the same arguments as in C apply and therefore player 1 will pass with a strictly positive probability.

If $\rho_1 > \frac{1}{24}$, then – even if player 2 is not sufficiently reciprocal – player 1 will pass in C with a probability high enough to make it materially profitable for player 2 to pass in B. The same arguments as above (for player 2 in B) show that player 1 passes in A with strictly positive probability. ■

8.11 A Game With No Reciprocity Equilibrium

A game where a reciprocity equilibrium may not exist is shown in Figure 5.

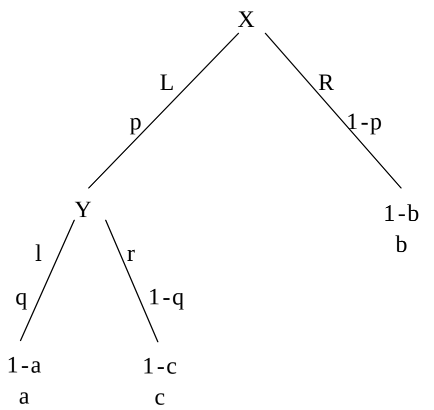

Figure 5: Game tree of a two person game that may not have a reciprocity equilibrium.

Assume that $\frac{1}{2} < a < b < c$ holds. In this game, all possible payoff shares result in an inequity in favor of player 2. Therefore, player 1 is always kind. In node Y player 1's kindness is intentional in this game if and only if $q''a + (1 - q'')c > b$. For instance, if $q'' = 1$, the kindness is not intentional. As a consequence, the reciprocal response will be weak, i.e., the second mover chooses a low q. If, on the other hand, $q'' = 0$, the kindness is intentional and a high q will be chosen. Hence, an equilibrium must lie somewhere in between. However, due to the discontinuity of function Ω, we can find parameters for which there is no reciprocity equilibrium, as claimed in the following proposition.

Proposition 13 *If* $\frac{1}{2b-1} \leq \rho_2 < \frac{1}{(2b-1)\varepsilon_2}$, *no reciprocity equilibrium exists in the game presented in Figure 5.*

Proof :
We show that there is no q that can be part of an equilibrium:
The utility of player 2 in the end nodes Ll and Lr is:

$$
\begin{aligned}
U_{2Ll} &= a + \rho_2 \vartheta_1(Y)(q''(a - (1-a) + (1-q'')(c - (1-c))(1-a) \\
U_{2Lr} &= a + \rho_2 \vartheta_1(Y)(q''(a - (1-a) + (1-q'')(c - (1-c))(1-c)
\end{aligned}
$$

We set:

$$\begin{aligned} U_{2Ll} - U_{2Lr} &= (c-a) + \rho_2 \vartheta_1(Y)(q''(2a-2c) + (2c-1)(a-c)) \\ &= (c-a)\left[1 - \rho_2 \vartheta_1(Y)(2c-1-q''(2a-2c))\right] \end{aligned}$$

We define:

$$q''_{crit} = \frac{c-\frac{1}{2}}{c-a} - \frac{1}{\rho_2 \vartheta_1(Y) 2(c-a)}$$

This is the solution of the equation $U_{2Ll} - U_{2Lr} = 0$. Player 2 will choose $q = 0$ if $q'' > q''_{crit}$ and $q = 1$ if $q'' < q''_{crit}$. We show that this is impossible for the parameters given in the proposition.

Case 1: If $q''a + (1-q'')c > b$ holds, there is full intention, i.e., $\vartheta_1(Y) = 1$. The inequality $q''a+(1-q'')c > b$ is equivalent to $q'' < \frac{c-b}{c-a}$. Furthermore, $\frac{1}{2b-1} < \rho_2$ holds by assumptions. Hence, we get $q_{crit} \geq \frac{c-\frac{1}{2}}{c-a} - \frac{1}{\frac{1}{2b-1}2(c-a)} = \frac{c-\frac{1}{2}}{c-a} - \frac{b-\frac{1}{2}}{c-a} = \frac{c-b}{c-a} > q''$.

As we have seen above this can only be the case if $q = q'' = 1$ and therefore, $q_{crit} > 1$. But by definition of a, b and c the inequality $\frac{c-b}{c-a} < 1$ holds.

Case 2: If $q''a+(1-q'')c \leq b$ holds, no intentions are involved and $\vartheta_1(Y) = \varepsilon_2$ holds. The inequality $q''a+(1-q'')c \leq b$ is equivalent to $q'' \geq \frac{c-b}{c-a}$. Furthermore, $\frac{1}{(2b-1)\varepsilon_2} > \rho_2$ holds by assumptions and we get $q_{crit} < \frac{c-\frac{1}{2}}{c-a} - \frac{1}{\frac{1}{2b-1}2\varepsilon_2(c-a)} = \frac{c-b}{c-a} \leq \frac{c-b}{c-a} \leq q''$. As we have seen above, this can only be the case if $q = q'' = 0$ and therefore, $q_{crit} \leq 0$. But by definition of a, b and c the inequality $\frac{c-b}{c-a} > 0$ holds. ■

Figure 6 shows the equilibrium choice of q dependent on ρ_2. As one can see, there is a range (between $1\frac{2}{3}$ and $8\frac{1}{3}$) where there is no equilibrium.

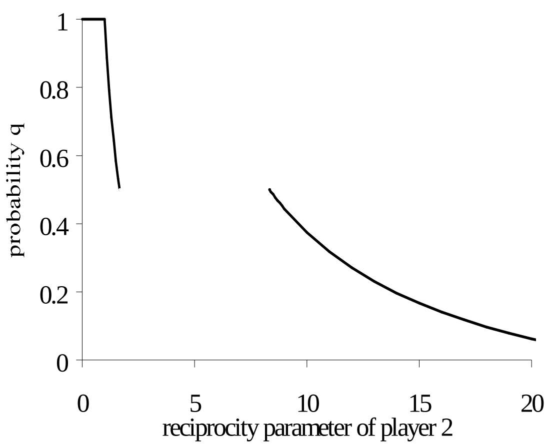

Figure 6: Equilibrium choice of q dependent on ρ_2 for the given parameters $a = 1$, $b = 0.8$, $c = 0.6$, $\varepsilon_2 = 0.2$.